Geometry and Topology: Team

(1) Let M be a surface in $\mathbb{R}^3$. Suppose for each point of M, there exist two families of geodesics which intersect at a constant angle. Prove that M has constant zero Gaussian curvature.

(2) Prove that two closed minimal hypersurfaces in S^n must intersect each other.

(3) Let M be a compact closed hypersurface in $\mathbb{R}^{n+1}$. Prove that M is an n-sphere if M has constant mean curvature and nonnegative Ricci curvature.

(4) Let $X \in \mathfrak{k} := Lie(K)$ be a real vector field on a compact connected smooth manifold M with an effective action of a compact real Lie group K. By choosing a K-invariant real symplectic form ω on M, assume $f \in C^\infty(M)_{\mathbb{R}}$ is such that

$$df = i_X \omega$$

Show that the value $\max_M f - \min_M f$ is independent of the choice of ω as far as ω defines the same de Rham cohomology class $[\omega]$.